

# CURRENT ELECTRICITY

## 1. ELECTRIC CURRENT

(a) Time rate of flow of charge through a cross section area is called **Current**.

$$I_{av} = \frac{\Delta q}{\Delta t} \text{ and instantaneous current } i = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

(b) Direction of current is along the direction of flow of positive charge or opposite to the direction of flow of negative charge. But the current is a scalar quantity.

$$\begin{array}{ccc} \longrightarrow i & & \longleftarrow i \\ q \oplus \longrightarrow \text{velocity} & & q \ominus \longrightarrow \text{velocity} \end{array}$$

SI unit of current is ampere and

$$1 \text{ Ampere} = 1 \text{ coulomb/sec}$$

$$1 \text{ coulomb/sec} = 1 \text{ A}$$

## 2. CONDUCTOR :

In some materials, the outer electrons of each atoms or molecules are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.

## 3. INSULATOR

Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

## 4. SEMICONDUCTOR

In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

## 5. ELECTRIC CURRENT IN A CONDUCTOR

In absence of potential difference across a conductor no net current flows through a cross section. When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) start drifting in a direction i.e. opposite to electric field with average drift velocity. If electrons are moving with velocity  $V$ ,  $A$  is area of cross section and  $n$  is number of free electrons per unit volume then,

$$I = nAeV.$$

$$V_d = \frac{\lambda}{\tau}, \text{ where } \lambda \text{ is mean free path and } \tau \text{ is relaxation time.}$$

$$V_d = \frac{1}{2} \left( \frac{eE}{m} \right) \tau^2 = \frac{1}{2} \frac{eE}{m} \tau, \text{ where } e \text{ is electronic charge, } E \text{ is electric field, } m \text{ is mass of electron}$$

Due to this drift, net current in metals is given by;

$$i = neAV_d$$

**Ex. 1** Find free electrons per unit volume in a metallic wire of density  $10^4 \text{ kg/m}^3$ , atomic mass number 100 and number of free electron per atom is one.

**Sol.** Number of free charge particle per unit volume ( $n$ ) =  $\frac{\text{total free charge particle}}{\text{total volume}}$

$\therefore$  Number. of free electron per atom means  
total free electrons = total number of atoms.

$$= \frac{N_A}{M_W} \times M$$

$$\text{So } n = \frac{\frac{N_A}{M_W} \times M}{V} = \frac{N_A}{M_W} \times d = \frac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}}$$

$$n = 6.023 \times 10^{28}$$

**Ex. 2** Find the approximate total distance travelled by an electron into the time-interval in which its displacement is one meter along the wire.

**Sol.** time =  $\frac{\text{displacement}}{\text{drift velocity}} = \frac{S}{V_d}$

$$\therefore V_d = 1 \text{ mm/s} = 10^{-3} \text{ m/s}$$

$$S = 1$$

$$\text{time} = \frac{1}{10^{-3}} = 10^3 \text{ s}$$

distance travelled = speed  $\times$  time

$$\therefore \text{speed} = 10^6 \text{ m/s}$$

$$\text{So required distance} = 10^6 \times 10^3 \text{ m}$$

$$= 10^9 \text{ m}$$

## 6. CURRENT DENSITY

Current density, a vector, at a point have magnitude equal to current per unit normal area at that point and direction is along the direction of the current at that point.

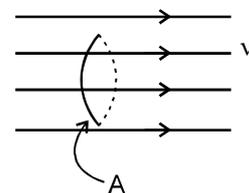
$$\vec{J} = \frac{di}{ds} \vec{n}$$

$$\text{so } di = \vec{J} \cdot d\vec{s}$$

Current is flux of current density.

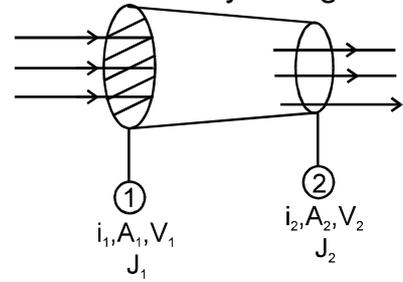
*Due to principle of conservation of charge:*

Charge entering at one end of a conductor = charge leaving at the other end, so current does not change with change in cross section and conductor remains uncharged when current flows through it.



**Q. 1** Current is flowing from a cylindrical conductor of non-uniform cross section area if  $A_1 > A_2$  then find relation between

(a)  $i_1$  and  $i_2$



(b)  $j_1$  and  $j_2$

(c)  $v_1$  and  $v_2$

where  $i$  is current,  $j$  is current density and  $V$  is drift velocity.

**Ans.**  $i_1 = i_2, V_1 < V_2, J_1 < J_2$

## 7. ELECTRICAL RESISTANCE

The property of a substance by virtue of that it opposes the flow of electric current through it is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

We have 
$$i = neAV_d = neA \left( \frac{eE}{2m} \right) \tau = \left( \frac{ne^2\tau}{2m} \right) AE$$

If  $V$  is potential difference applied to the ends of a conductor of length  $\ell$  then

$$E = \frac{V}{\ell}$$

so 
$$i = \left( \frac{ne^2\tau}{2m} \right) \left( \frac{A}{\ell} \right) V = \left( \frac{A}{\rho\ell} \right) V = V/R \Rightarrow V = IR$$

$\rho$  is called resistivity (it is also called specific resistance), and  $\rho = \frac{2m}{ne^2\tau} = \frac{1}{\sigma}$ ,  $\sigma$  is called conductivity.

Therefore current in conductors is proportional to potential difference applied across its ends. This is **Ohm's Law**. Units:  $R \rightarrow \text{ohm}(\Omega)$ ,  $\rho \rightarrow \text{ohm-meter}(\Omega\text{-m})$  also called siemens,  $\sigma \rightarrow \Omega^{-1}\text{m}^{-1}$ .

### 7.1 Dependence of Resistance on various factors

$$R = \rho \frac{\ell}{A} = \frac{2m}{ne^2\tau} \cdot \frac{\ell}{A}$$

Therefore  $R$  depends as

(1)  $\propto \ell$       (2)  $\propto \frac{1}{A}$       (3)  $\propto \frac{1}{n}$      $\propto \frac{1}{\tau}$

(4) and in metals  $\tau$  decreases as  $T$  increases  $\Rightarrow R$  also increases.

#### Results

(a) On stretching a wire (volume constant)

If length of wire is changed then 
$$\frac{R_1}{R_2} = \frac{\ell_1^2}{\ell_2^2}$$

If radius of cross section is changed then 
$$\frac{R_1}{R_2} = \frac{r_2^4}{r_1^4}$$
, where  $R_1$  and  $R_2$  are initial and final resistances and  $\ell_1, \ell_2$ , are initial and final lengths and  $r_1$  and  $r_2$  initial and final radii respectively.

(b) Effect of percentage change in length of wire

$$\frac{R_2}{R_1} = \frac{\ell^2 \left[1 + \frac{x}{100}\right]^2}{\ell^2} \text{ where } \ell - \text{original length and } x - \% \text{ increment}$$

if x is quite small (say < 5%) then % change in R is

$$\frac{R_2 - R_1}{R_1} \times 100 = \frac{\left(1 + \frac{x}{100}\right)^2 - 1}{1} \cong 2x\%$$

**Ex. 3** If a wire is stretched to double its length, find the new resistance if original resistance of the wire was R.

**Sol.** As we know that  $R = \frac{\rho \ell}{A}$

in case  $R' = \frac{\rho \ell'}{A'}$

$\ell' = 2\ell$     $A' \ell' = A\ell$    by conservation of (volume of the wire remains constant)

$$A' = \frac{A}{2}$$

$$R' = \frac{\rho \times 2\ell}{A/2} = 4 \frac{\rho \ell}{A} = 4R$$

**Ex. 4** The wire is stretched to increase the length by 1% find the percentage change in the Resistance.

**Sol.** As we know that

$$\therefore R = \frac{\rho \ell}{A}$$

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A} \text{ and } \frac{\Delta \ell}{\ell} = - \frac{\Delta A}{A}$$

$$\frac{\Delta R}{R} = 0 + 1 + 1 + 1$$

$$= 2$$

Hence percentage increase in the Resistance = 2%

**Note :** Above method is applicable when % change is very small.

**Dependence of Resistance on Temperature :**

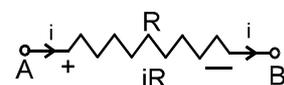
The resistance of most conductors and all pure metals increases with temperature, but there are a few in which resistance decreases with temperature. If  $R_0$  and R be the resistance of a conductor at  $0^\circ\text{C}$  and at  $\theta^\circ\text{C}$ , then it is found that  $R = R_0 (1 + \alpha\theta)$ .

Where  $\alpha$  is temperature coefficient of resistance.

The unit of  $\alpha$  is  $\text{K}^{-1}$  or  $^\circ\text{C}^{-1}$ .

**Electric current in resistance**

In a resistor current flows from high potential to low potential



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 High potential is represented by positive (+) sign and low potential is represented by negative (-) sign.

$$V_A - V_B = iR$$

If  $V_1 > V_2$

then current will flow from A to B

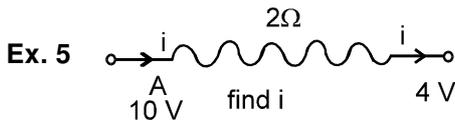


and  $i = \frac{V_1 - V_2}{R}$

If  $V_1 < V_2$

then current will go from A to B

and  $i = \frac{V_2 - V_1}{R}$

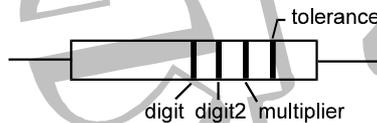


**Sol.**  $V_A - V_B = i \times R$

$i = \frac{6}{2} = 3A$  Ans.

### 7.2 Colour code for Resistors

Resistors of different values are commercially available. To make a resistor, carbon with a suitable binding agent is molded into a cylinder. Wire leads are



attached to the cylinder and the entire resistor is encased in a ceramic or plastic jacket. The two leads connect the resistor to a circuit such as those for radios, amplifiers etc. The value of the resistance is indicated by four coloured-bands, marked on the surface of the cylinder (figure). The meanings of the four positions of the bands are shown in figure and the meanings of different colours are given in table.

**Table : Resistance codes (resistance given in ohm)**

Colour	Digit	Multiplier	Tolerance
Black	0	1	
Brown	1	10	
Red	2	10 <sup>2</sup>	
Orange	3	10 <sup>3</sup>	
Yellow	4	10 <sup>4</sup>	
Green	5	10 <sup>5</sup>	
Blue	6	10 <sup>6</sup>	
Violet	7	10 <sup>7</sup>	
Gray	8	10 <sup>8</sup>	
White	9	10 <sup>9</sup>	
Gold		0.1	5%
Silver		0.01	10%

For example, suppose the colours on the resistor shown in figure are brown yellow, green and gold as read from left to right. Using table, the resistance is

$$(14 \times 10^5 \pm 5\%)W = (1.4 \pm 0.07) M\Omega.$$

Brown	Yellow	Green	Gold
1	4	10	5%

Sometimes, the tolerance band is missing from the code so that there are only three bands. This means the tolerance is 20%.

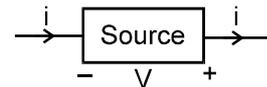
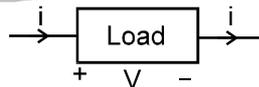
## 8. ELECTRICAL POWER

Energy liberated per second in a device is called its power. The electrical power  $P$  delivered or consumed by an electrical device is given by  $P = VI$ , where  $V$  = Potential difference across the device and  $I$  = Current.

If the current enters the higher potential point of the device then electric power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).

$$\text{Power} = \frac{V \cdot dq}{dt}$$

$$= VI \quad \quad \quad \mathbf{P = V I}$$



If power is constant then energy =  $P t$

If power is variable then

$$\text{Energy} = \int pdt$$

Power consumed by a resistor

$$P = I^2R = VI = \frac{V^2}{R}$$

When a current is passed through a resistor energy is wasted in overcoming the resistance of the wire. This energy is converted into heat.

$$W = VIt = I^2Rt = \frac{V^2}{R}t$$

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The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for T second is given by:

$$H = I^2 RT \text{ Joule} = \frac{I^2 RT}{4.2} \text{ Calorie}$$

1 unit of electrical energy = 1 Kilowatt hour = 1 KWh =  $3.6 \times 10^6$  Joule.

**Ex. 6** If bulb rating is 100 watt and 220 V then determine

- (a) Resistance of filament
- (b) Current through filament
- (c) If bulb operate at 110 volt power supply then find power consume by bulb.

**Sol.** Bulb rating is 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consume is 100 W

Here  $V = 220$

$P = 100$

$$\frac{V^2}{R} = 100$$

So  $R = 484 \Omega$

Since Resistance depends only on material hence it is constant for bulb

$$I = \frac{V}{R} = \frac{220}{22 \times 22} = \frac{5}{11} \text{ Amp.}$$

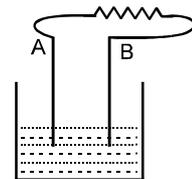
power consumed at 110 V

$$\begin{aligned} \therefore \text{power consumed} &= \frac{110 \times 110}{480} \\ &= 25 \text{ W} \end{aligned}$$

## 9. BATTERY (CELL)

A battery is a device which maintains a potential difference across its two terminals A and B. Dry cells, secondary cells, generator and thermocouple are the devices used for producing potential difference in an electric circuit. Arrangement of cell or battery is shown in figure.

Electrolyte provides continuity for current.



**Ex. 7** What is the meaning of 10 Amp. hr ?

**Sol.** It means if the 10 A current is withdrawn then the battery will work for 1 hour.

10 Amp  $\longrightarrow$  1 hr

1 Amp  $\longrightarrow$  10 hr

$\frac{1}{2}$  Amp  $\longrightarrow$  20 hr

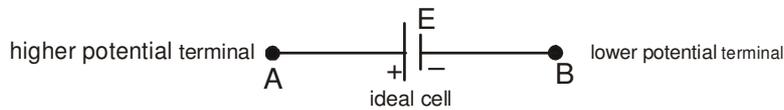
## 10. ELECTROMOTIVE FORCE : (E.M.F.)

**Definition I :** Electromotive force is the capability of the system to make the charge flow.

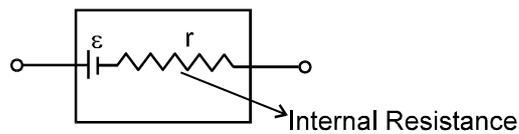
**Definition II :** It is the work done by the battery for the flow of 1 coulomb charge from lower potential terminal to higher potential terminal inside the battery.

### 10.1 Representation for battery

**Ideal cell :** Cell in which there is no heating effect.



**Non ideal cell :** Cell in which there is heating effect inside due to opposition to the current flow internally



Non ideal cell.

**Case I :**

**Battery acting as a source (or battery is discharging)**

$$V_A - V_B = \epsilon - ir$$

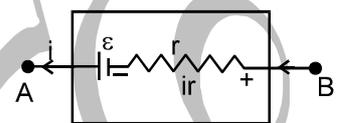
$V_A - V_B \Rightarrow$  it is also called terminal voltage.

The rate at which the chemical energy of the cell is consumed =  $\epsilon i$

The rate at which heat is generated inside the battery or cell =  $i^2 r$

electric power output =  $\epsilon i - i^2 r$

$$= (\epsilon - ir) i$$



**Case II :**

**Battery acting as a load (or battery charging).**

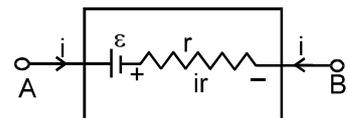
$$V_A - V_B = \epsilon + ir$$

the rate at which chemical energy stored in the cell =  $\epsilon i$

thermal power inside the cell =  $i^2 r$

electric power input =  $\epsilon i + i^2 r = (\epsilon + ir) i$

$$= (V_A - V_B) i$$



**Definition III :**

**Electromotive force** of a cell is equal to potential difference between its terminals when no current is passing through the circuit.

**Case III :**

When cell is in open circuit

$i = 0$  as resistance of open circuit is  $\infty$ .

So  $V = \epsilon$ , so open circuit terminal voltage difference is equal to emf of the cell.

**Case IV :**

When cell is short circuited

$i = \frac{\epsilon}{r}$  and  $V = 0$ , short circuit current of a cell is maximum.

**Note :** The potential at all points of a wire of zero resistance will be same.

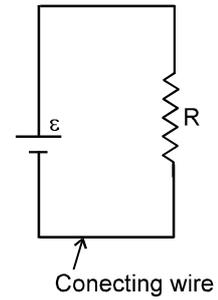
## 11 RELATIVE POTENTIAL

While solving an electric circuit it is convenient to choose a reference point and assigning its voltage as zero, then all other potentials are measured with respect to this point. This point is also called the common point.

**Q. 2** Find the power consumed by the resistance. The connecting wire have negligible resistance for calculation.

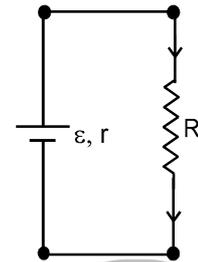
**Ans.** Since the cell is ideal and the connecting wires are of zero resistance so the whole EMF will appear as potential difference across R :

$\therefore P = \frac{\epsilon^2}{R}$  we notice from this formula that any amount of power can be taken from cell by adjusting the value of R.



**Ex. 8** In the given electric circuit find

- current
- power output
- relation between  $r$  and  $R$  so that the electric power output (that means power given to  $R$ ) is maximum.
- value of maximum power output.
- plot graph between power and resistance of load
- From graph we see that for a given power output there exists two values of external resistance, prove that the product of these resistances equals  $r^2$ .
- what is the efficiency of the cell when it is used to supply maximum power.



**Sol.** (a) In the circuit shown if we assume that potential at A is zero then potential at B is  $\epsilon - ir$ . Now since the connecting wires are of zero resistance

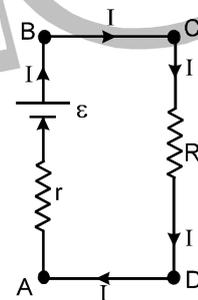
$$\begin{aligned} \therefore V_D &= V_A = 0 \\ V_C &= V_B = \epsilon - ir \end{aligned}$$

Now current through wire is also  $I$

( $\because$  its in series with the cell).

$$\therefore I = \frac{V_C - V_D}{R} = \frac{(\epsilon - Ir) - 0}{R}$$

$$\text{Current } I = \frac{\epsilon}{r + R}$$



**Note :** After the learning the concept of series combination we will be able to calculate the current directly

(b) Power output

$$P = I^2 R = \frac{\epsilon^2}{(r + R)^2} \cdot R$$

$$(c) \frac{dP}{dR} = \frac{\epsilon^2}{(r + R)^2} - \frac{2\epsilon^2 R}{(r + R)^3}$$

for maximum power supply

$$\frac{dp}{dR} = 0$$

$$r + R - 2R = 0$$

$$\mathbf{r = R}$$

Here for maximum power output outer Resistance should be equal to internal resistance

(d)  $P_{\max} = \frac{\epsilon^2}{4r}$

(e) Graph between 'P' and R  
maximum power output at  $R = r$

$$P_{\max} = \frac{\epsilon^2}{4r}$$

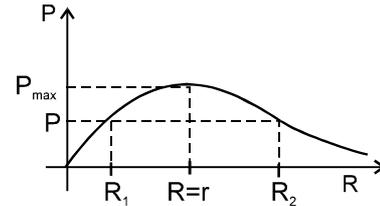
(f)  $i = \frac{\epsilon}{r + R}$

Power output

$$P = \frac{\epsilon^2 R}{(r + R)^2}$$

$$P(r^2 + 2rR + R^2) = \epsilon^2 R$$

$$R^2 + (2r - \frac{\epsilon^2}{P})R + r^2 = 0$$



∴ above quadratic equation in R has two roots  $R_1$  and  $R_2$  for given values of  $\epsilon$ , P and r such that  $R_1 R_2 = r^2$  (product of roots)

$$r^2 = R_1 R_2$$

(g) Power of battery spent

$$= \frac{\epsilon^2}{(r + r)^2} \cdot 2r$$

$$= \frac{\epsilon^2}{2r}$$

$$\text{power (output)} = \left(\frac{\epsilon}{r + r}\right)^2 \times r = \frac{\epsilon^2}{4r}$$

$$\text{Efficiency} = \frac{\text{power output}}{\text{total power spent by cell}} = \frac{\frac{\epsilon^2}{4r} \times 100}{\frac{\epsilon^2}{2r}} = \frac{1}{2} \times 100 = 50\%$$

**Ex. 9** Shown in the figure

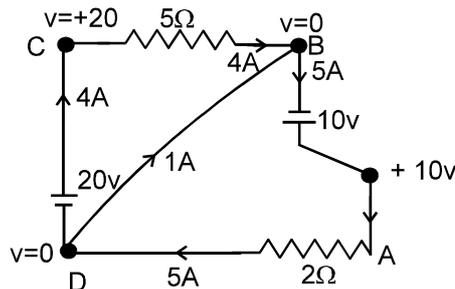
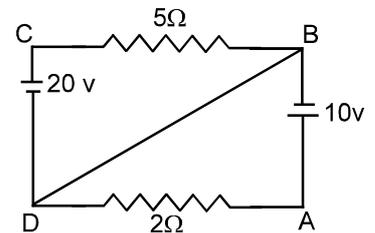
Find out the current in the wire BD

**Sol.** Let at point D potential = 0 and write the potential of other points then

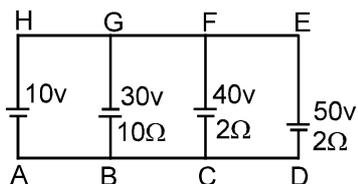
$$\text{current in wire AD} = \frac{10}{2} = 5 \text{ A from A to D} \quad \text{current in wire CB} = \frac{20}{5} = 4 \text{ A}$$

from C to B

∴ current in wire BD = 1 A from D to B



**Ex. 10** Find the current in each wire



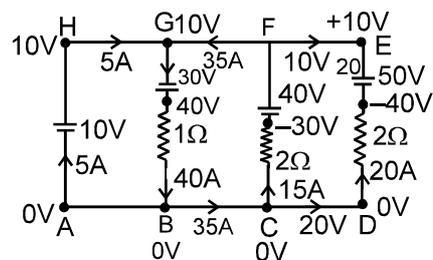
**Sol.** let potential at point A is 0 volt then potential of other points is shown in figure.

$$\text{current in BG} = \frac{40 - 0}{1} = 40 \text{ A from G to B}$$

$$\text{current in FC} = \frac{0 - (-30)}{2} = 15 \text{ A from C to K}$$

$$\text{current in DE} = \frac{0 - (-40)}{2} = 20 \text{ A from D to E}$$

$$\text{current in wire AH} = 40 - 35 = 5 \text{ A from A to H}$$



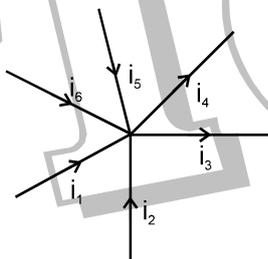
## 12. KIRCHHOFF'S LAWS

### 12-1- Kirchhoff's Current Law (Junction law)

This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a point of the circuit is zero" or total currents entering a junction equals total current leaving the junction.

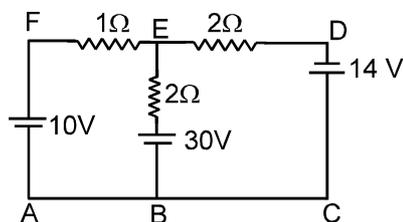
$$\Sigma I_{in} = \Sigma I_{out} \text{ It is also known as KCL (Kirchhoff's current law).}$$

**Ex. 11** Find relation in between current  $i_1, i_2, i_3, i_4, i_5$  and  $i_6$ .

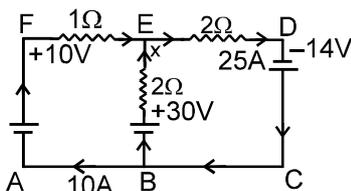


**Sol.**  $i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$

**Ex. 12** Find the current in each wire



**Sol.**



Let potential at point B = 0. Then potential at other points are mentioned.

∴ Potential at E is not known numerically.

Let potential at E = x

Now applying kirchhoff's current law at junction E. (This can be applied at any other junction also).

$$\frac{x-10}{1} + \frac{x-30}{2} + \frac{x+14}{2} = 0$$

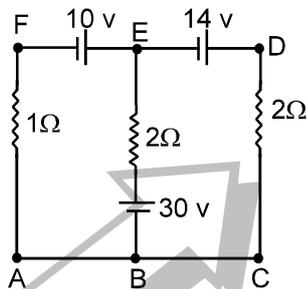
$$4x = 36 \quad \Rightarrow \quad x = 9$$

$$\text{Current in EF} = \frac{10-9}{1} = 1 \text{ A from F to E}$$

$$\text{Current in BE} = \frac{30-9}{2} = 10.5 \text{ A from B to E}$$

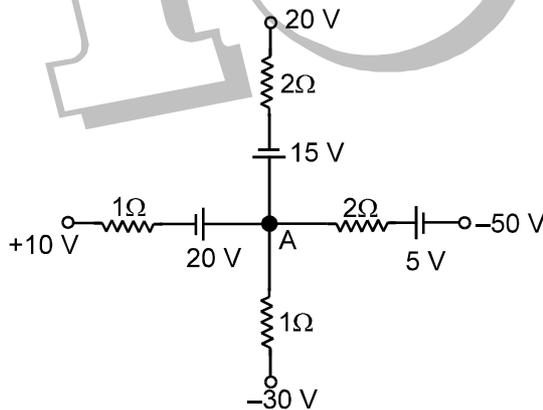
$$\text{Current in DE} = \frac{1-(-14)}{2} = 7.5 \text{ A from E to D}$$

**Q. 3** Find the current in each wire :



**Ans.** current in AFE = 1 A from A to E  
 current in EB = 10.5 A from B to E  
 current in ED = 7.5 A from E to D

**Ex. 13** Find the potential at point A



**Sol.** Let potential at A = x, applying kirchhoff current law at junction A

$$\frac{x-20-10}{1} + \frac{x-15-20}{2} + \frac{x+45}{2} + \frac{x+30}{1} = 0$$

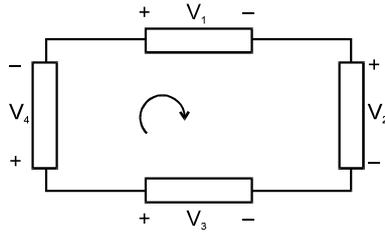
$$\Rightarrow \frac{2x-60+x-35+x+45+2x+60}{2} = 0$$

$$\Rightarrow 6x + 10 = 0 \quad \Rightarrow \quad x = -5/3$$

$$\text{Potential at A} = \frac{-5}{3} \text{ V}$$

**12.2 Kirchhoff's Voltage Law (Loop law)**

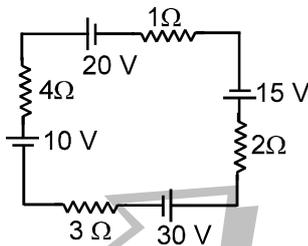
"The algebraic sum of all the potential differences along a closed loop is zero.  $\sum IR + \sum EMF = 0$ ". The closed loop can be traversed in any direction. While traversing a loop if potential increases, put a positive sign in expression and if potential decreases put a negative sign.



$-V_1 - V_2 + V_3 - V_4 = 0$ . Boxes may contain resistor or battery or any other element (linear or nonlinear).

It is also known as **KVL**

**Ex.14** Find current in the circuit



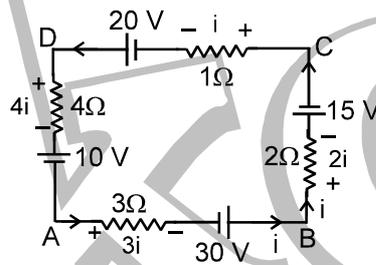
**Sol.** ∴ all the elements are connected in series  
 ∴ current is all of them will be same  
 let current =  $i$

Applying kirchhoff voltage law in ABCDA loop

$$10 + 4i - 20 + i + 15 + 2i - 30 + 3i = 0$$

$$10i = 25$$

$$i = 2.5 \text{ A}$$



**Ex. 15** Find the current in each wire applying only kirchhoff voltage law

**Sol.** Applying kirchhoff voltage law in loop ABEFA

$$i + 30 + 2(i_1 + i_2) - 10 = 0$$

$$3i_1 + 2i_2 + 20 = 0 \quad \text{----- (i)}$$

Applying kirchhoff voltage law in BCDEB

$$+ 30 + 2(i_1 + i_2) + 50 + 2i_2 = 0$$

$$4i_2 + 2i_1 + 80 = 0$$

$$2i_2 + i_1 + 40 = 0 \quad \text{----- (ii)}$$

Solving (i) and (ii)

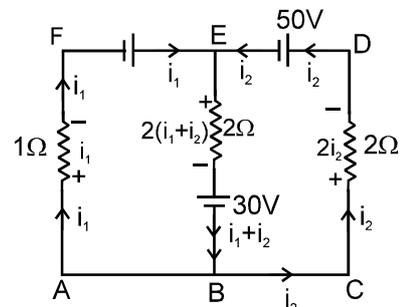
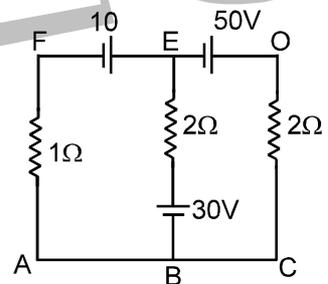
$$3[-40 - 2i_2] + 2i_2 + 20 = 0$$

$$-120 - 4i_2 + 20 = 0$$

$$i_2 = -25 \text{ A and } i_1 = 10 \text{ A}$$

$$\therefore -i_1 + i_2 = -15 \text{ A}$$

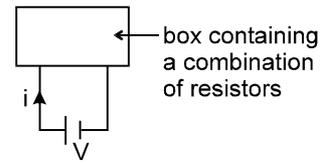
current in wire AF = 10 A from A to E  
 current in wire EB = 15 A from B to E  
 current in wire DE = 25 A from D to C.



### 13. COMBINATION OF RESISTANCES :

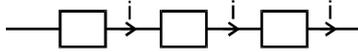
A number of resistances can be connected and all the complicated combinations can be reduced to two different types, namely series and parallel.

The equivalent resistance of a combination is defined as  $R_{eq} = \frac{V}{i}$

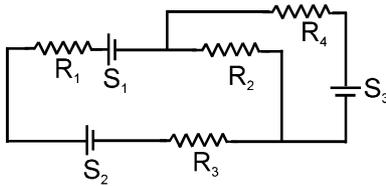


#### 13.1 Resistances in Series:

When the resistances (or any type of elements) are connected end to end then they are said to be in series. The current through each element is same.

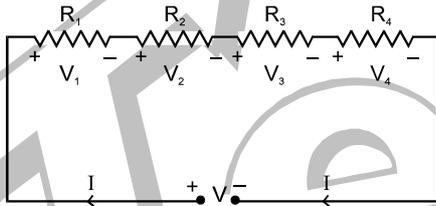


**Ex. 16** Which electrical elements are connected in series.



**Sol.** Here  $S_1, S_2, R_1, R_3$  connected in series and  $R_4, S_3$  connected in different series

#### Equivalent of Resistors :



The effective resistance appearing across the battery (or between the terminals A and B) is

$$R = R_1 + R_2 + R_3 + \dots + R_n \quad (\text{this means } R_{eq} \text{ is greater than any resistor) and}$$

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

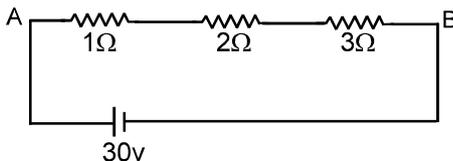
The potential difference across a resistor is proportional to the resistance. Power in each resistors is also proportional to the resistance

$$\therefore V = IR \text{ and } P = I^2R$$

where I is same through any of the resistor.

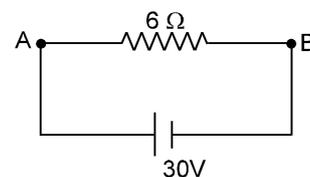
$$V_1 = \frac{R_1}{R_1 + R_2 + \dots + R_n} V ; V_2 = \frac{R_2}{R_1 + R_2 + \dots + R_n} V ; \text{ etc}$$

**Ex. 17** Find the current in the circuit



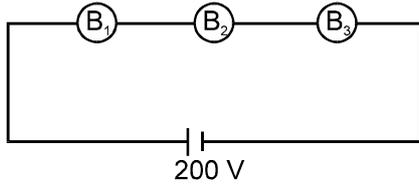
**Sol.**  $R_{eq} = 1 + 2 + 3 = 6 \Omega$  the given circuit is equivalent to

$$\text{current } i = \frac{V}{R_{eq}} = \frac{30}{6} = 5A$$



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**Ex.18** In the figure shown  $B_1, B_2$  and  $B_3$  are three bulbs rated as (200V, 50 W), (200V, 100W) and (200 V, 25W) respectively. Find the current through each bulb and which bulb will give more light?



**Sol.**  $R_1 = \frac{(200)^2}{50}$  ;  $R_2 = \frac{(200)^2}{100}$  ;  $R_3 = \frac{(200)^2}{25}$

the current following through each bulb is =  $\frac{200}{R_1 + R_2 + R_3}$

$$= \frac{200}{(200)^2 \left[ \frac{2+1+4}{100} \right]}$$

$$= \frac{100}{200 \times 7} = \frac{1}{14} \text{ A}$$

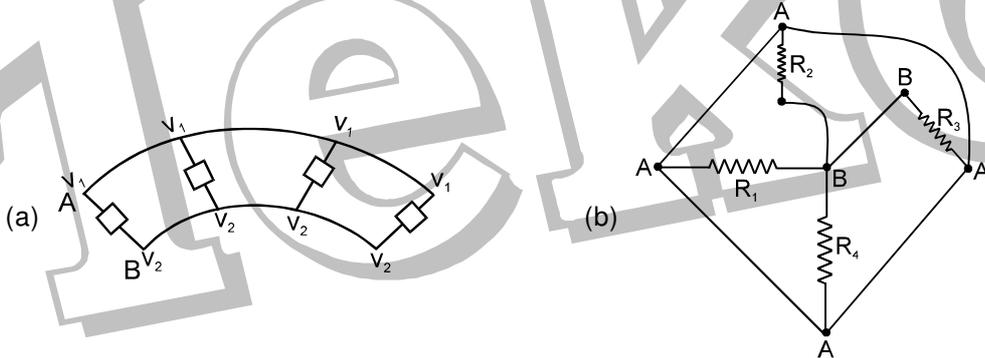
Since  $R_3 > R_1 > R_2$

$\therefore$  Power consumed by bulb =  $i^2R$   $\therefore$  if the resistance is of higher value then it will give more light.

$\therefore$  Here Bulb  $B_3$  will give more light.

**13.2 Resistances in Parallel :**

A parallel circuit of resistors is one in which the same voltage is applied across all the components in a parallel grouping of resistors  $R_1, R_2, R_3, \dots, R_n$ .



In the figure (a) and (b) all the resistors are connected between points A and B so they are in parallel.

**Equivalent resistance**

Applying kirchhoff's junction law at point P

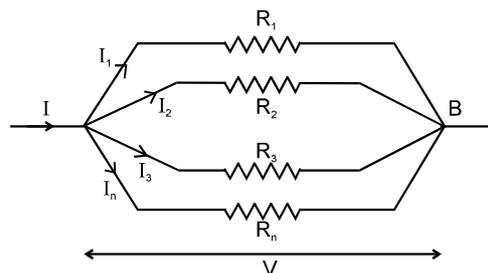
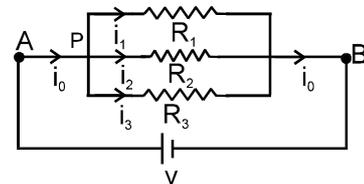
$$i_0 = i_1 + i_2 + i_3$$

Therefore,  $\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

in general,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$



**Conclusions: (about parallel combination)**

- (a) Potential difference across each resistor is same.
- (b)  $I = I_1 + I_2 + I_3 + \dots + I_n$ .
- (c) Effective resistance (R) then  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$ . ( $R_{eq}$  is less than each resistors).
- (d) Current in different resistors is inversely proportional to the resistance.

$$I_1 : I_2 : \dots : I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \dots : \frac{1}{R_n}$$

$$I_1 = \frac{G_1}{G_1 + G_2 + \dots + G_n} I, I_2 = \frac{G_2}{G_1 + G_2 + \dots + G_n} I, \text{ etc.}$$

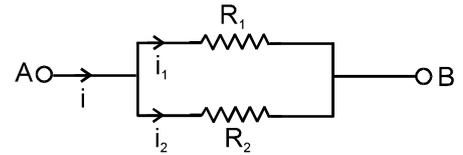
where  $G = \frac{1}{R}$  = Conductance of a resistor. [Its unit is  $\Omega^{-1}$  or  $\text{mho}$ ]

**Ex. 19** When two resistors are in parallel combination then determine  $i_1$  and  $i_2$  ?

**Sol.**  $\therefore i_1 R_1 = i_2 R_2$

or  $\frac{i_1}{i_2} = \frac{R_2}{R_1}$

$$i_1 = \frac{R_2 i}{R_1 + R_2} \Rightarrow i_2 = \frac{R_1 i}{R_1 + R_2}$$



**Note :** Remember this law of  $i \propto \frac{1}{R}$  in the resistors connected in parallel. It can be used in problems.

**Ex. 20** Find current passing through the battery and each resistor.

**Sol. Method (I)**

It is easy to see that potential difference across each resistor is 30 V.

$\therefore$  current is each resistors are  $\frac{30}{2} = 15 \text{ A}$ ,  $\frac{30}{3} = 10 \text{ A}$  and  $\frac{30}{6} = 5 \text{ A}$

$\therefore$  Current through battery is =  $15 + 10 + 5 = 30 \text{ A}$ .

**Method (II)**

By ohm's law  $i = \frac{V}{R_{eq}}$

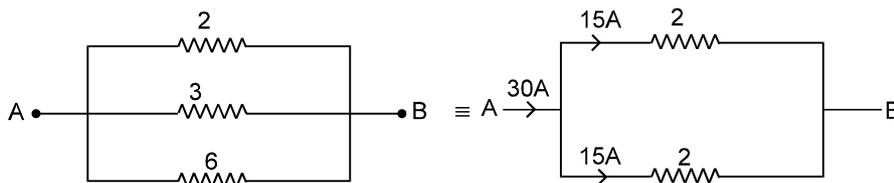
$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

$$= 1 \Omega$$

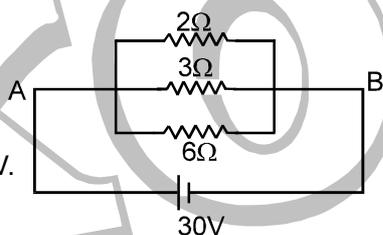
$$R_{eq} = 1 \Omega$$

$$i = \frac{30}{1} = 30 \text{ A}$$

Now distribute this current in the resistors in their inverse ratio.



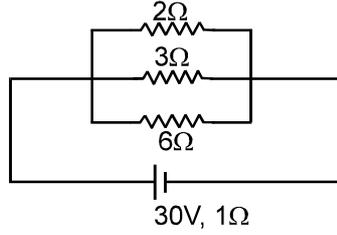
Current total in 3  $\Omega$  and 6  $\Omega$  is 15 A it will be divided as 10 A and 5 A.



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**Note :** The method (I) is better. But you will not find such an easy case every where.

**Ex. 21** Find current which is passing through battery.



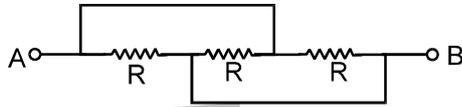
**Sol.** Here potential difference across each resistor is not 30 V

∴ battery has internal resistance here the concept of combination of resistors is useful.

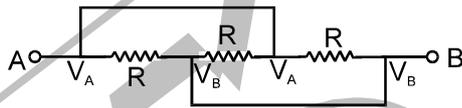
$$R_{eq} = 1 + 1 = 2 \Omega$$

$$i = \frac{30}{2} = 15 \text{ A.}$$

**Ex. 22** Find equivalent Resistance



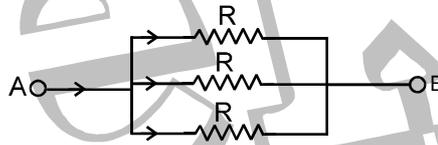
**Sol.**



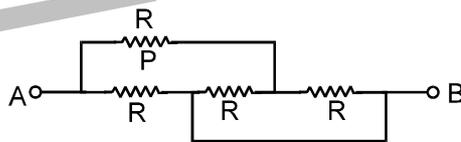
Here all the Resistance are connected between the terminals A and B

Modified circuit is

So  $R_{eq} = \frac{R}{3}$



**Ex. 23** Find the current in Resistance P if voltage supply between A and B is V volts

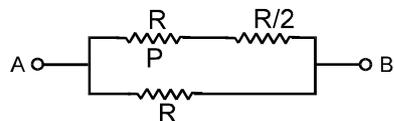
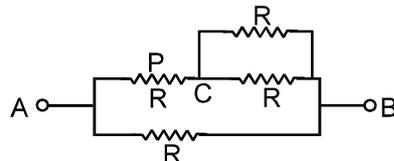
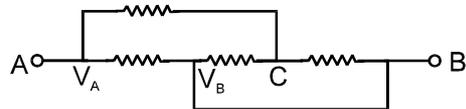


**Sol.**

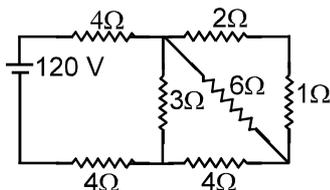
$$R_{eq} = \frac{13R}{5}$$

$$I = \frac{5V}{3R} \quad \text{Modified circuit}$$

$$\begin{aligned} \text{Current in P} &= \frac{R \times \frac{5V}{3R}}{1.5R + R} \\ &= \frac{2V}{3R} \end{aligned}$$



**Ex. 24** Find the current in  $2\Omega$  resistance



**Sol.**  $2\Omega, 1\Omega$  in series =  $3\Omega$

$$3\Omega, 6\Omega \text{ in parallel} = \frac{18}{9} = 2\Omega$$

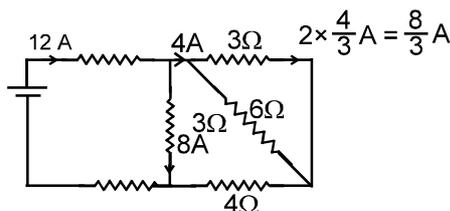
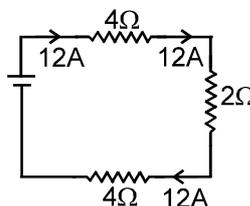
$$2\Omega, 4\Omega \text{ in series} = 6\Omega$$

$$6\Omega, 3\Omega \text{ in parallel} = 2\Omega$$

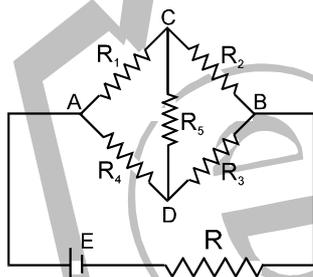
$$R_{eq} = 4 + 4 + 2 = 10\Omega$$

$$i = \frac{120}{10} = 12A$$

$$\text{So current in } 2\Omega \text{ Resistance} = \frac{8}{3} A$$



#### 14. WHEATSTONE NETWORK : (4 TERMINAL NETWORK)



The arrangement as shown in figure, is known as Wheat stone bridge

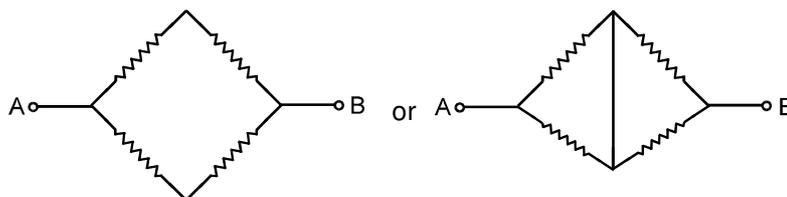
Here there are four terminals in which except two all are connected to each other through resistive elements.

In this circuit if  $R_1 R_3 = R_2 R_4$  then  $V_C = V_D$  and current in  $R_5 = 0$  this is called balance point or null point

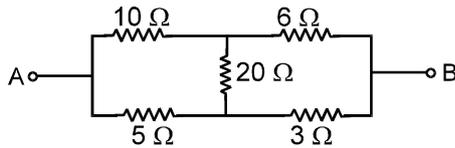
When current through the galvanometer is zero (null point or balance point)  $\frac{P}{Q} = \frac{R}{S}$ , then  $PS = QR \Rightarrow$

Here in this case products of opposite arms are equal. Potential difference between C and D at null point is zero. The null point is not affected by resistance  $R_5$ , E and R. It is not affected even if the positions of G and E are interchanged.

hence, here the circuit can be assumed to be following,



**Ex.25** Find equivalent resistance of the circuit between the terminals A and B.

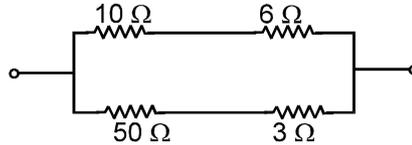


**Sol.** Since the given circuit is wheat stone bridge and it is in balance condition.

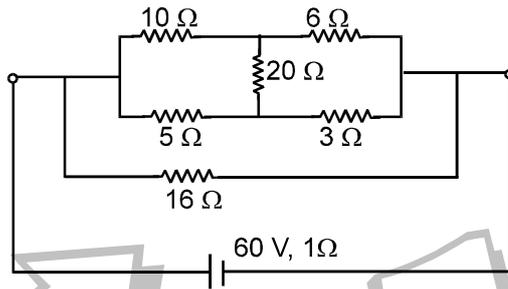
$$\therefore 10 \times 3 = 30 = 6 \times 5$$

hence this is equivalent to

$$R_{eq} = \frac{16 \times 8}{16 + 8} = \frac{16}{3} \Omega$$



**Ex.26**



Find (a) Equivalent resistance (b) and current in each resistance

**Sol.** (a)  $R_{eq} = \left( \frac{1}{16} + \frac{1}{8} + \frac{1}{16} \right)^{-1} + 1 = 5 \Omega$

(b)  $i = \frac{60}{4 + 1} = 12 \text{ A}$

Hence 12 A will flow through the cell.

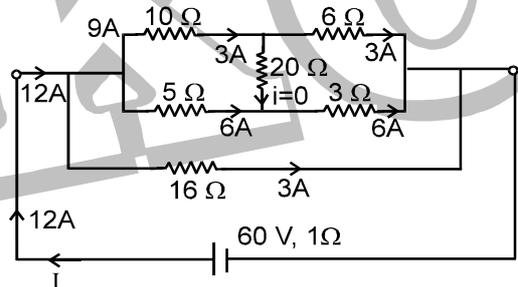
By using current distribution law.

Current in resistance  $10 \Omega$  and  $6 \Omega = 3 \text{ A}$

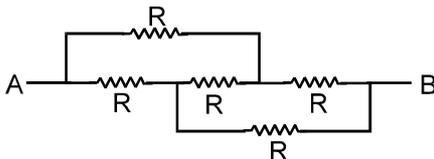
Current in resistance  $5 \Omega$  and  $3 \Omega = 6 \text{ A}$

Current in resistance  $20 \Omega = 0$

Current in resistance  $16 \Omega = 3 \text{ A}$

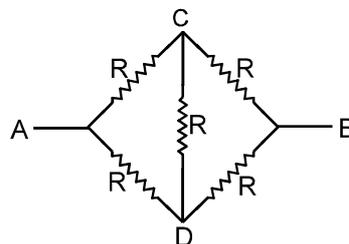


**Ex.27** Find the equivalent resistance between A and B



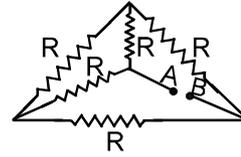
**Sol.** This arrangement can be modified as shown in figure since it is balanced wheat stone bridge

$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$



**Q.4** Find the equivalent Resistance between A and B

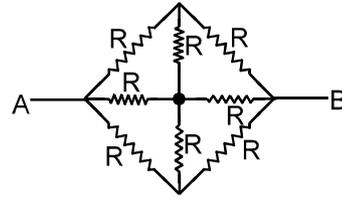
**Ans.**  $R_{eq} = R$



**15. SYMMETRICAL CIRCUITS :**

Some circuits can be modified to have simpler solution by using symmetry if they are solved by traditional method of KVL and KCL then it would take much time.

**Ex.28** Find the equivalent Resistance between A and B



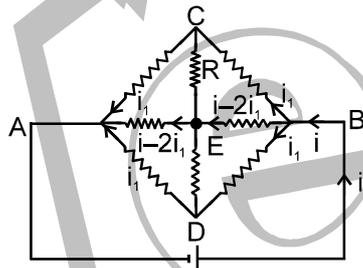
**Sol. I Method :**

Here no two resistors appear to be in series or parallel no wheatstone bridge here. This circuit will be

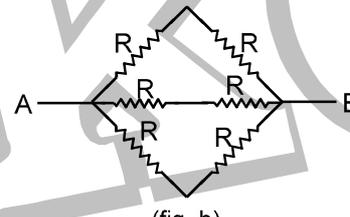
solve by using  $R_{eq} = \frac{V}{I}$ . The branches AC and AD are symmetrical

∴ current through them will be same.

The circuit is also similar from left side and right side current distribution while entering through B and an exiting from A will be same. Using all these facts the currents are as shown in the figure. It is clear that current in resistor between C and E is 0 and also in ED is 0. It's equivalent is shown in figure (b)



(fig. a)



(fig. b)

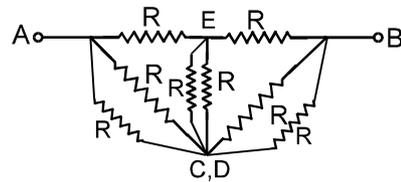
$$R_{eq} = \frac{2R}{3}$$

**II Method**

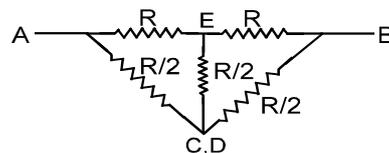
∴ The potential difference in R between (B, C) and between (B,D.) is same  $V_c = V_D$

Hence the point C and D are same hence circuit can be simplified as

This called folding.



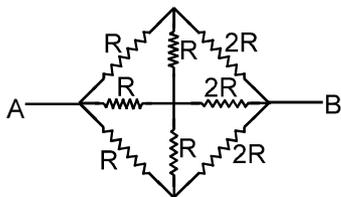
Now, it is Balanced wheatstone bridge



$$R_{eq} = \frac{2R \times R}{2R + R} = \frac{2R}{3}$$

**Note :** In II Method it is not necessary to know the currents in CA and DA.

**Ex.29** Find the equivalent Resistance between A and B



**Sol.** In this case the circuit has symmetry in the two branches AC and AD at the input  
 $\therefore$  current in them are same but from input and from exit the circuit is not similar  
 ( $\because$  on left R and on right 2R)

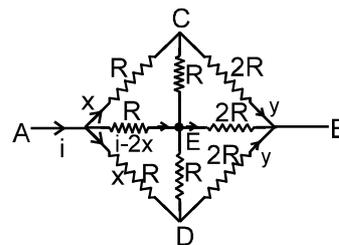
$\therefore$  on both sides the distribution of current will not be similar.

Here  $V_c = V_d$

hence C and D are same point

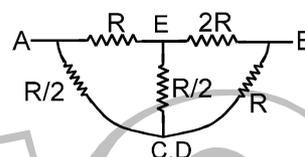
the circuit can be simplified that

Now it is balanced wheat stone bridge

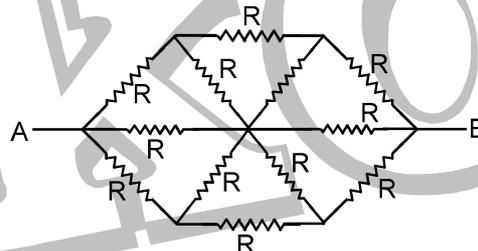


$$R_{eq} = \frac{3R \times \frac{3R}{2}}{3R + \frac{3R}{2}}$$

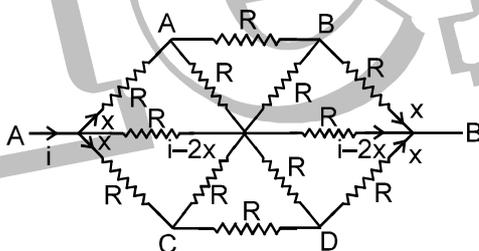
$$= \frac{\frac{9}{2}R}{\frac{9}{2}} = R.$$



**Ex. 30** Find the equivalent Resistance between A and B



**Sol.**



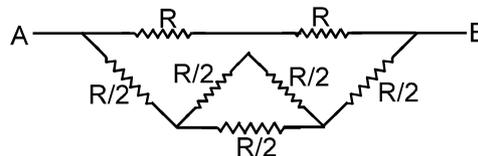
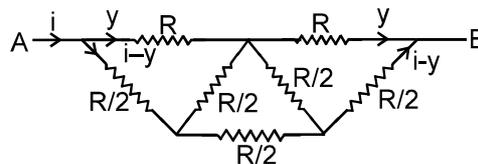
Here  $V_A = V_C$  and  $V_B = V_D$

Here the circuit can be simplified as

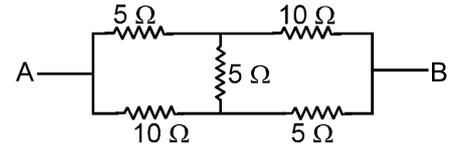
this circuit can be simplified as

$$R_{eq} = \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}}$$

$$= \frac{4R}{5} \quad \text{Ans.}$$

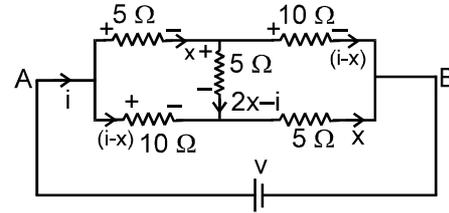


**Ex. 31** Find the equivalent Resistance between A and B



**Sol.** It is wheat stone bridge but not balanced. No series parallel connections. But similar values on input side and output. Here we see that even after using symmetry the circuit does not reduce to series parallel combination as in previous examples.

∴ applying kirchoff voltage law  
 $v - 10(i - x) - 5x = 0$   
 $v - 10i + 5x = 0 \dots(1)$   
 $10(i - x) - 5(2x - i) - 5x = 0$   
 $10i - 10x - 10x + 5i - 5x = 0$   
 $15i - 25x = 0$



$$x = \frac{15}{25} i$$

$$5x = 3i \dots(2)$$

Using (2) and (1)

$$\therefore v - 10i + 3i = 0$$

$$\frac{v}{I} = 7\Omega$$

$$R_{eq} = 7\Omega$$

**Ans.**

## 16. GROUPING OF CELLS

### 16.1 Cells in Series :

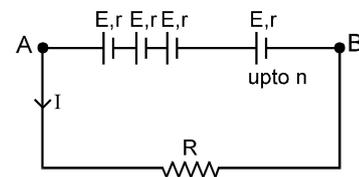


Equivalent EMF  $E_{eq} = E_1 + E_2 + \dots + E_n$  [write EMF's with polarity]

Equivalent internal resistance  $r_{eq} = r_1 + r_2 + r_3 + r_4 + \dots + r_n$

If n cells each of emf E, arranged in series and if r is internal resistance of each cell, then total

emf = n E so current in the circuit  $I = \frac{nE}{R + nr}$



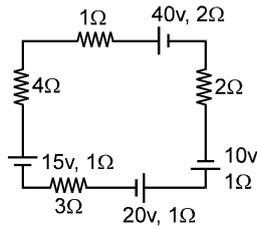
If  $nr \ll R$  then  $I = \frac{nE}{R} \rightarrow$  Series combination is advantageous.

If  $nr \gg R$  then  $I = \frac{E}{r} \rightarrow$  Series combination is not advantageous.

**Note** - If polarity of m cells is reversed, then equivalent emf = (n-2m)E while the equivalent resistance is still nr+R, so current in R will be

$$i = \frac{(n-2m)E}{nr + R}$$

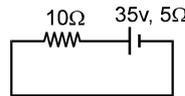
**Ex.32** Find the current in the loop.



**Sol.** The given circuit can be simplified as

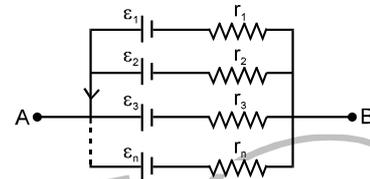
$$i = \frac{35}{10+5} = \frac{35}{15} = \frac{7}{3} \text{ A}$$

$$I = \frac{7}{3} \text{ A}$$



**16.2 Cells in Parallel:**

$$E_{eq} = \frac{\frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2} + \dots + \frac{\epsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}} \quad [\text{Use emf's with polarity}]$$

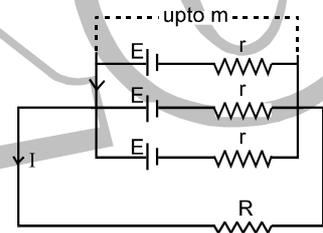


$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$$

If m cells each of emf E and internal resistance r be connected in parallel and if this combination is connected to an external resistance then the emf of the circuit = E.

Internal resistance of the circuit =  $\frac{r}{m}$ .

$$\text{and } I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$$



If  $mR \ll r$ ;  $I = \frac{mE}{r}$  → Parallel combination is advantageous.

If  $mR \gg r$ ;  $I = \frac{E}{R}$  → Parallel combination is not advantageous.

**16.3 Cells in Multiple Arc :**

$mn$  = number of identical cells.

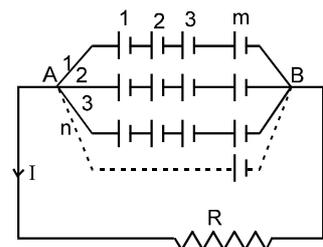
$n$  = number of rows

$m$  = number of cells in each row.

The combination of cells is equivalent to single cell of

$$\text{emf} = mE \quad \text{and} \quad \text{internal resistance} = \frac{mr}{n}$$

$$\text{Current } I = \frac{mE}{R + \frac{mr}{n}}$$



For maximum current  $nR = mr$  or

$$R = \frac{mr}{n} = \text{internal resistance of the equivalent battery.}$$

$$I_{\max} = \frac{nE}{2r} = \frac{mE}{2R}.$$

## 17. GALVANOMETER

Galvanometer is represented as follow :



Inside the galvanometer there is coil (kept in a uniform magnetic field produced by magnetic poles) which rotates due to the magnetic field when current is passed. There is a spiral type of spring as shown in figure.



When coil rotates the spring is twisted and it exerts an opposing torque on the coil.

There is a resistive torque also against motion to damp the motion. Finally in equilibrium

$$\tau_{\text{magnetic}} = \tau_{\text{spring}}$$

$$\Rightarrow BINA \sin \theta = C\phi$$

But by making the magnetic field radial  $\theta = 90^\circ$ .

$$\therefore BINA = C\phi$$

$$I \propto \phi$$

here  $B =$  magnetic field

$I =$  Current

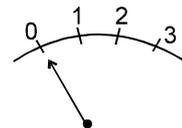
$N =$  Number of turns

$A =$  Area of the coil

$C =$  torsional constant

$\phi =$  angle rotate by coil.

A linear scale is obtained the marking on the galvanometer are proportionate.

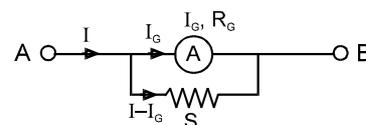
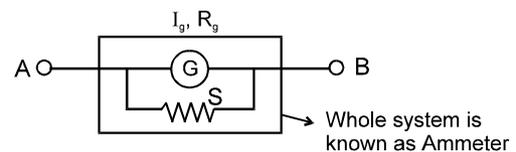
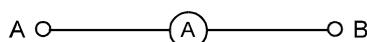


The galvanometer coil has some resistance represented by  $R_g$ . It is of the order few ohms. Its also has a maximum capacity to carry a current known as  $I_g$ .  $I_g$  is also the current required for full scale deflection. This galvanometer is called moving coil galvanometer.

## 18. AMMETER

A shunt (small resistance) is connected in parallel with galvanometer to convert into ammeter; An ideal ammeter has zero resistance

Ammeter is represented as follow -



If maximum value of current to be measured by ammeter is  $I$  then

$$I_g \cdot R_g = (I - I_g)S$$

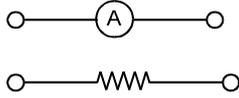
$$S = \frac{I_G \cdot R_G}{I - I_G}$$

$$S = \frac{I_G \times R_G}{I} \quad \text{when } I \gg I_G.$$

where  $I$  = Maximum current that can be measured using the given ammeter.

For measuring the current the ammeter is connected in series.

For calculation it is simply a resistance



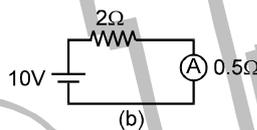
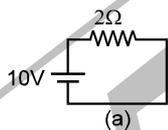
Resistance of ammeter

$$R_A = \frac{R_G \cdot S}{R_G + S}$$

for  $S \ll R_G$

$$\Rightarrow R_A = S$$

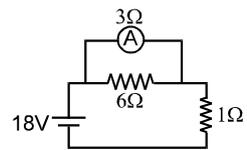
**Ex. 33** Find the current in the circuit for larger range of ammeter 'S' should be smaller (a) and (b).



**Sol.** In A  $I = \frac{10}{2} = 5A$   
 In B  $I = \frac{10}{2.5} = 4A$

here we see that the due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

**Ex. 34** Find the reading of ammeter Is this the current through  $6 \Omega$  ?



**Sol.**  $R_{eq} = \frac{3 \times 6}{3 + 6} + 1 = 3 \Omega$

Current through battery

$$I = \frac{18}{3} = 6 A$$

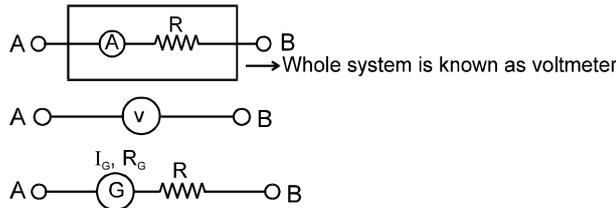
So, current through ammeter =  $6 \times \frac{6}{9} = 4 A$

No, it is not the current through the  $6 \Omega$  resistor.

**Note :** Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.

### 19. VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.



For maximum potential difference

$$V = I_G \cdot R_s + I_G R_G$$

$$R_s = \frac{V}{I_G} - R_G$$

$$\text{If } R_G \ll R_s \Rightarrow R_s \approx \frac{V}{I_G}$$

For measuring the potential difference a voltmeter is connected across that element. (parallel to the that element it measures the potential difference that appears between terminals 'A' and 'B'.)

For calculation it is simply a resistance



Resistance of voltmeter  $R_v = R_G + R_s \approx R_s$

$$I_g = \frac{V_o}{R_g + R}$$

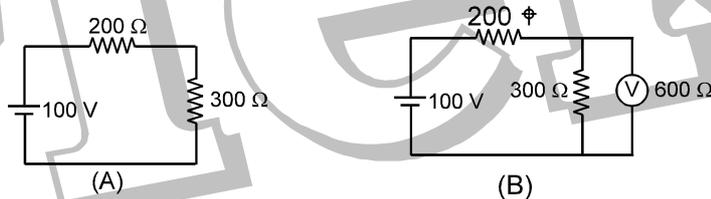
$R \rightarrow \infty \Rightarrow$  Ideal voltmeter.

A good voltmeter has high value of resistance.

Ideal voltmeter  $\rightarrow$  which has high value of resistance.

**Note :** For calculation purposes the current through the ideal voltmeter is zero.

**Ex.35** Find potential difference across the resistance  $300 \Omega$  in A and B.



**Sol.** In (A) : Potential difference =  $\frac{100}{200 + 300} \times 300 = 60$  volt

In (B) : Potential difference =  $\frac{100}{200 + \frac{300 \times 600}{300 + 600}} \times \frac{300 \times 600}{300 + 600} = 50$  volt

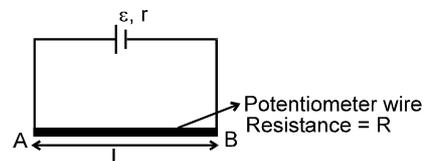
We see that by connected voltmeter the voltage which was to be measured has changed. Such voltmeters are not good. If its resistance had been very large than  $300 \Omega$  then it would have not affected the voltage by much amount.

### 20. POTENTIOMETER

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire. Any potential difference which is less then the potential difference maintained across the potentiometer wire can be measured using this.

The wire should have high resistivity and low expansion coefficient for example. Manganin or, Constantine wire etc.

$$I = \frac{\epsilon}{r + R}$$



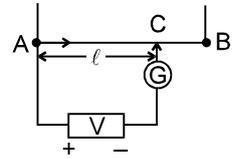
$$V_A - V_B = \frac{\epsilon}{R+r} \cdot R$$

Potential gradient (x) → Potential difference per unit length of wire

$$x = \frac{V_A - V_B}{L} = \frac{\epsilon}{R+r} \cdot \frac{R}{L}$$

**Ex.36** How to measure an unknown voltage using potentiometer.

**Sol.** The unknown voltage V is connected across the potentiometer wire as shown in figure. The positive terminal of the unknown voltage is kept on the same side as of the source of the top most battery. When reading of galvanometer is zero then we say that the meter is balanced. In that condition  $V = x \cdot l$ .



**20.1 Application of potentiometer**

(a) **To find emf of unknown cell and compare emf of two cells.**

In case I,

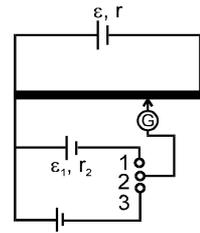
In figure (3) is joint to (1) then balance length =  $l_1$   
 $\epsilon_1 = x l_1$  ....(1)

in case II,

In figure (3) is joint to (2) then balance length =  $l_2$   
 $\epsilon_2 = x l_2$  ....(2)

$$\frac{\epsilon_1}{\epsilon_2} = \frac{l_1}{l_2}$$

If any one of  $\epsilon_1$  or  $\epsilon_2$  is known the other can be found. If x is known then both  $\epsilon_1$  and  $\epsilon_2$  can be found



(b) **To find current if resistance is known**

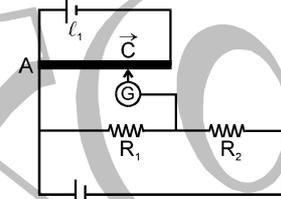
$$V_A - V_C = x l_1$$

$$I R_1 = x l_1$$

$$I = \frac{x l_1}{R_1}$$

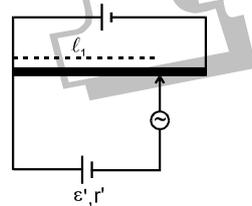
Similarly, we can find the value of  $R_2$  also.

Potentiometer is ideal voltmeter because it does not draw any current from circuit, at the balance point.



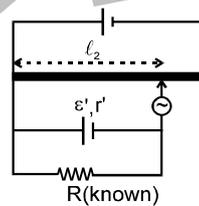
(c) **To find the internal resistance of cell.**

1<sup>st</sup> arrangement



by first arrangement  $\epsilon' = x l_1$  ....(1)

2<sup>nd</sup> arrangement



by second arrangement  $I R = x l_2$

$$I = \frac{x l_2}{R}, \quad \text{also } I = \frac{\epsilon'}{r'+R}$$

$$\therefore \frac{\epsilon'}{r'+R} = \frac{x l_2}{R}$$

$$\Rightarrow \frac{x l_1}{r'+R} = \frac{x l_2}{R}$$

$$r' = \left[ \frac{l_1 - l_2}{l_2} \right] R$$

(d) **Ammeter and voltmeter can be graduated by potentiometer.**

(e) **Ammeter and voltmeter can be calibrated by potentiometer.**

## 21. METRE BRIDGE (USE TO MEASURE UNKNOWN RESISTANCE)

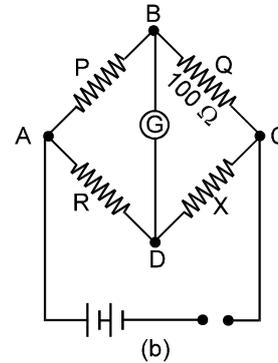
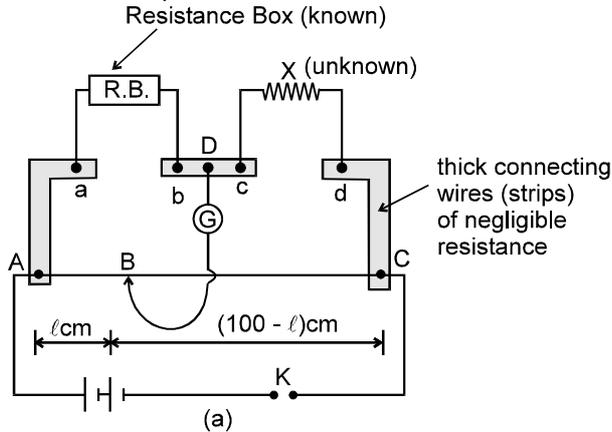
If  $AB = \ell$  cm, then  $BC = (100 - \ell)$  cm.

Resistance of the wire between A and B  $R \propto \ell$

[  $\because$  Specific resistance  $\rho$  and cross-sectional area  $A$  are the same for the whole of the wire ]

$$\text{or } R = \sigma \ell \quad \dots(1)$$

where  $\sigma$  is resistance per cm of wire.



Similarly, if  $Q$  is resistance of the wire between B and C, then

$$Q \propto 100 - \ell$$

$$\therefore Q = \sigma(100 - \ell) \quad \dots(2)$$

Dividing (1) by (2), 
$$\frac{P}{Q} = \frac{\ell}{100 - \ell}$$

Applying the condition for balanced Wheatstone bridge, we get

$$R Q = P X$$

$$\therefore x = R \frac{Q}{P}$$

$$\text{or } X = \frac{100 - \ell}{\ell} R$$

Since  $R$  and  $\ell$  are known, therefore, the value of  $X$  can be calculated.

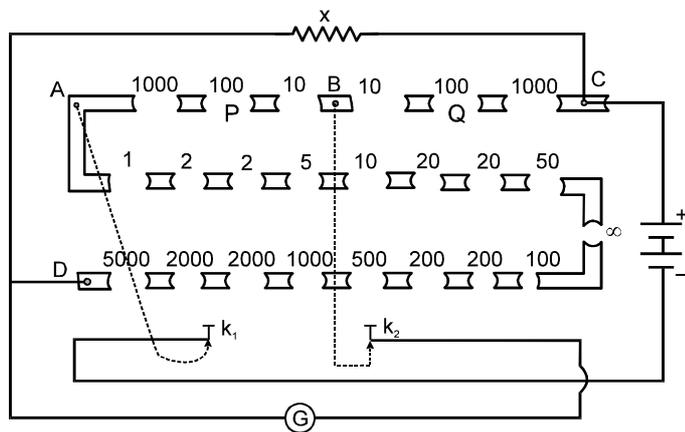
**Note :** For better accuracy,  $R$  is so adjusted that  $\ell$  lies between 40 cm and 60 cm.

## 22. Post-office Box

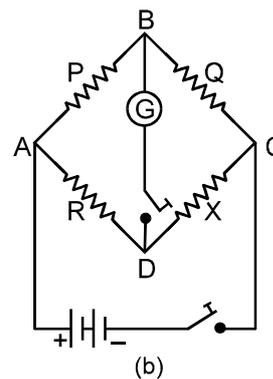
**Introduction.** It is so named because its shape is like a box and it was originally designed to determine the resistances of electric cables and telegraph wires. It was used in post offices to determine the resistance of transmission lines.

**Construction.** A post office box is a compact form of Wheatstone bridge with the help of which we can measure the value of the unknown resistance correctly up to 2nd decimal place, i.e., up to 1/100th of an ohm correctly. Two types of post office box are available - plug type and dial type. In the plug-type instrument shown in figure (a), each of the arms AB and BC contains three resistances of 10, 100 and 1000 ohm. These arms are called the ratio arms. While the resistance  $P$  can be introduced in the arm AB, the resistance  $Q$  can be introduced in the arm BC. The third arm AD, called the resistance arm, is a complete resistance box containing resistances from 1  $\Omega$  to 5,000  $\Omega$ . In this arm, the resistance  $R$  is introduced by taking out plugs of suitable values. The unknown resistance  $X$  constitutes the fourth arm CD. Thus, the four arms AB, BC, CD and AD are in fact the four arms of the Wheatstone bridge (figure (b)). Two tap keys  $K_1$  and  $K_2$  are also provided. While  $K_1$  is connected internally to the terminal A,  $K_2$  is connected internally to B. These internal connections are shown by dotted lines in figure (a).

A battery is connected between C and key  $K_1$  (battery key). A galvanometer is connected between D and key  $K_2$  (galvanometer key). Thus, the circuit is exactly the same as that shown in figure (b). It is always the battery key which is pressed first and then the galvanometer key. This is because a self-induced current is always set up in the circuit whenever the battery key is pressed or released. If we first press the galvanometer key, the balance point will be disturbed on account of induced current. If the battery key is pressed first, then the induced current becomes zero by the time the galvanometer key is pressed. So, the balance point is not affected.



(a)



(b)

**Working :** The working of the post office box involves broadly the following four steps :

- I. Keeping R zero, each of the resistances P and Q are made equal to 10 ohm by taking out suitable plugs from the arms AB and BC respectively. After pressing the battery key first and then the galvanometer key, the direction of deflection of the galvanometer coil is noted. Now, making R infinity, the direction of deflection is again noted. If the direction is opposite to that in the first case, then the connections are correct.
- II. Keeping both P and Q equal to 10Ω, the value of R is adjusted, beginning from 1Ω, till 1 Ω increase reverses the direction of deflection. The 'unknown' resistance clearly lies somewhere between the two final values of R.

$$\left[ X = R \frac{Q}{P} = R \frac{10}{10} = R \right]$$

As an illustration, suppose with 3Ω resistance in the arm AD, the deflection is towards left and with 4Ω, it is towards right. The unknown resistance lies between 3Ω and 3Ω.

- III. Making P 100 Ω and keeping Q 10 Ω, we again find those values of R between which direction of deflection is reversed. Clearly, the resistance in the arm AD will be 10 times the resistance X of the wire.

$$\left[ X = R \frac{Q}{P} = R \frac{10}{100} = \frac{R}{10} \right]$$

In the illustration considered in step II, the resistance in the arm AD will now lie between 30 Ω, and 30 Ω. So, in this step, we have to start adjusting R from 30 Ω onwards. If 32 Ω and 33 Ω are the two values of R which give opposite deflections, then the unknown resistance lies between 3.2 Ω and 3.3 Ω.

- IV. Now, P is made 1000 Ω and Q is kept at 10 Ω. The resistance in the arm AD will now be 100 times the 'unknown' resistance.

$$\left[ X = R \frac{10}{1000} = \frac{R}{100} \right]$$

In the illustration under consideration, the resistance in the arm AD will lie between 320 Ω and 330Ω. Suppose the deflection is to the right for 326 ohm, towards left for 324 ohm and zero deflection for 325Ω Then, the unknown resistance is 3.25 Ω.

The post office box method is a less accurate method for the determination of unknown resistance as compared to a metre bridge. This is due to the fact that it is not always possible to arrange resistance in the four arms to be of the same order. When the arms ratio is large, large resistance are required to be introduced in the arm R.

(CIE) posir.

## SUMMARY

- $I_{av} = \frac{\Delta q}{\Delta t}$  and  $i = \frac{dq}{dt}$
- $i = ne A V_d$
- $V = IR$
- $R = \frac{\rho \ell}{A}$
- Power  $P = VI$
- $P = I^2 R = \frac{V^2}{R}$
- Energy = power  $\times$  time
- The rate at which the chemical energy of the cell is consumed =  $Ei$
- The rate at which heat is generated inside the battery =  $i^2 r$
- Electric power output =  $(\epsilon - ir) i$
- Maximum power output when internal resistance = external resistance
- In series combination  $R = R_1 + R_2 + R_3 + \dots$
- In parallel combination  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$
- Cell in series
- $E_{eq} = \epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_n$  (write Emf's with polarity)
- $r_{eq} = r_1 + r_2 + r_3 + \dots$
- Cells in parallel
- $$E_{eq} = \frac{\frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2} + \dots + \frac{\epsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}}$$
 (Use proper sign before the EMFs)
- and  $\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$
- In Ammeter shunt (S) =  $\frac{I_G \times R_G}{I - I_G}$
- In voltmeter  $V = I_G R_s + I_G R_G$
- Potential gradient in potentiometer
- $$x = \frac{\epsilon}{R+r} \times \frac{R}{L}$$